Conformism and Diversity under Social Learning

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Abstract

When there are competing technologies or products with unknown payoffs which are adopted over time within a society, an important question is whether conformism or diversity will prevail. We use a learning model with local interactions to study this question.

We show that the structure of information flows within a society helps to determine whether conformism or diversity obtains. We find that if information is public then society conforms to a single technology in the long run. On the other hand, if society consists of smaller groups of individuals and interaction within groups is more intense as compared to interaction across the groups, then two technologies can coexist and diversity obtains, in the long run. Our analysis involves a novel application of the Law of the Iterated Logarithm.

Key Words: Social learning, local interactions, conformism/diversity, networks.

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1 Introduction

When payoffs from different products or technologies are unknown, agents use their own past experience as well as the experience of their neighbors to update their prior beliefs. This experience consists of actions and their corresponding outcomes. Given these beliefs, she chooses an action that maximizes expected utility. We study the evolution of agents' beliefs, actions and utilities.[2] Our focus is on the following question:

- Does the process of social learning lead, over time, to the dominance of a single technology or is there coexistence of different technologies?

In an earlier paper, Bala and Goyal (1993), we showed that if a society is *connected*[3] then *different* qualities cannot coexist in the long run: eventually one or the other is driven out. This does not address the issue of what happens when the technologies are of the same intrinsic quality. The present paper considers this situation.

Our principal finding is that if the technologies are of the same intrinsic quality, then their coexistence depends upon the nature of the *neighborhood structure* in a society. Specifically, if a society is divided into smaller sub-groups and there are more direct information links between agents within the same group as compared to links across different groups, then two technologies can coexist, in the long run, with positive probability (Proposition 2.3). This is true in a connected society and even if all agents start with the same prior beliefs! On the other hand, if every agent directly observes everyone else – a situation of uniform or global interaction – then conformism is certain to arise: all agents in the society will choose the same technology in the long run, with probability 1 (Proposition 2.4). We also discuss simulations which show that the likelihood of coexistence is positively related to the 'degree of integration' within the society.

[2]Examples of such environments include consumers learning about different brands, farmers learning about the productivity of improved seeds/pesticides and doctors learning about the efficacy of new treatments. Empirical work has documented the importance of learning from "others" in several contexts such as the adoption of crops (Ryan and Gross, 1943), the diffusion of patent drugs (Coleman, 1966) and the choice of agricultural techniques (Hagerstrand, 1969; Rogers, 1983).

[3]A society is said to be connected if for every pair of agents i and j, either j is a neighbor of i or there exist agents $i_1, i_2, \ldots, i_m$ (depending upon i and j) such that i_1 is a neighbor of i, i_2 is a neighbor of i_1 and so on, until j is a neighbor of i_m. In other words, in a connected society information can potentially get transmitted between any pair of agents over time.

Our analysis provides insights into empirically observed variations in medical practice across regions and countries. For example, Taylor (1979) reports that in the nineteen thirties, the frequency of tonsillectomy in some regions of England was one half of the national average while in other regions it was almost three times as high. Similarly, in the late sixties and seventies, tonsillectomy was six times more frequent in New England than in Sweden. Similar differences exist in several other fields of medical practice. Our results suggest that an important reason for these differences is the structure of information sources: doctors will generally have much closer contact with other doctors within the same region/country as compared to doctors in other countries.[4] This influences the flow of information and sustains different practices across countries/regions.

The result concerning coexistence is also interesting from a technical perspective, since it involves a novel application of the Law of the Iterated Logarithm. It also raises the possibility that this theorem may have more general applicability in analysing long-run dynamics in other models with local interactions.

Our paper is part of the literature on social learning.[5] The question of conformism and diversity is also addressed in Ellison and Fudenberg (1995). However, in their models agents use only currently available social information, such as recent popularity weighting and disregard historical data (including their own past experience) in making decisions. By contrast, in our model agents do use historical information directly in updating their beliefs. Moreover, the (boundedly) Bayesian updating rule agents employ precludes the use of popularity weighting. Ellison and Fudenberg study the possibilities of conformism and diversity under different levels of popularity weighting and sample sizes. On the other hand, we focus on the role of neighborhood structures. These differences suggest that our approach should be seen as complementary to their work.

[4] For example, they are more likely to attend the same conferences and read the same local medical journals.

[5] Early work is reported in Allen (1982). In recent years, considerable attention has been devoted to this subject. This work includes Bala and Goyal (1993, 1995), Banerjee (1992), Bikchandani, Hirshleifer, and Welch (1992), Bolton and Harris (1992), Ellison and Fudenberg (1993, 1995) and Vives (1992), among others.

2 The Model and Results

There are two actions x_1 and x_2 each of two possible types, High (H) and Low (L). An action of High type yields Bernoulli payoffs $y = 1$ (success) and 0 (failure) with probabilities π and $1 - \pi$ respectively, where $\pi \in (1/2, 1)$ is a parameter. An action of the Low type yields payoffs 1 and 0 but with probabilities $1 - \pi$ and π. Let Θ denotes the set of states of nature. From the above description we have $\Theta = \{(H, H), (H, L), (L, H), (L, L)\}$, corresponding to the types of the two actions. We assume that the true state is $\theta \in \Theta$.

The society consists of a finite set of agents denoted by N. Agents in the society do not know the true type of each action. Each agent has a prior belief (probability distribution) on Θ. We assume that the types of actions are *independent* of each other: hence, the beliefs of an agent are given by $\xi = (\mu, \nu) \in [0, 1]^2$ where μ is the probability that x_1 is a High type and ν the probability that x_2 is a High type.

We now specify the nature of the social learning process. For each agent $i \in N$, we let $N(i) \subset N$ be the set of 'neighbors' of agent i. If j is a neighbor of i, then in each period $t = 1, 2, \ldots$, agent i observes the action chosen by j and the payoff (1 or 0) that j receives from his action. We make the convention that $i \in N(i)$ for each $i \in N$. We shall also suppose that the society is *connected*, i.e. for all i and j either $j \in N(i)$ or there exists a set of agents $\{i_1, \ldots i_q\} \subset N$ (depending upon i and j) such that $i_1 \in N(i), i_2 \in N(i_1), \ldots, j \in N(i_q)$.

At the beginning of period 1, agent i has a prior belief $\xi_{i,1} = (\mu_{i,1}, \nu_{i,1})$. We assume that $\xi_{i,1} \in (0, 1)^2$ for each $i \in N$. In period 1, agent i chooses an action which maximizes one-period expected value with respect to the prior $\xi_{i,1}$. Since actions are symmetric, this means

$$x_1 \text{ is chosen } \Rightarrow \mu_{i,1} > \nu_{i,1}, \quad x_2 \text{ is chosen } \Rightarrow \mu_{i,1} > \nu_{i,1}. \tag{1}$$

Ties are assumed to be broken in an arbitrary way, provided the choice is a measurable function of the prior belief. After choosing her action she gets to observe the (stochastic) outcome. She also gets to observe the action (x_1 or x_2) and outcome (0 and 1) of every agent $j \in N(i)$. In period $t = 2$, agent i computes her posterior belief $\xi_{i,2} = (\mu_{i,2}, \nu_{i,2})$ based on the experiences of the agents in $N(i)$ as follows. Suppose she observes $n \geq 0$ trials and S_n 'successes' with action x_1 and $m \geq 0$ trials and R_m 'successes' with action x_2 among her neighbors. From Bayes' rule, her posterior belief is given by:

$$\mu_{i,2} = \frac{\mu_{i,1}\pi^{S_n}(1-\pi)^{n-S_n}}{\mu_{i,1}\pi^{S_n}(1-\pi)^{n-S_n} + (1-\mu_{i,1})\pi^{n-S_n}(1-\pi)^{S_n}} = \frac{\mu_{i,1}}{\mu_{i,1} + (1-\mu_{i,1})p^{2S_n-n}}$$

$$\nu_{i,2} = \frac{\nu_{i,1}\pi^{R_m}(1-\pi)^{m-R_m}}{\nu_{i,1}\pi^{R_m}(1-\pi)^{m-R_m} + (1-\nu_{i,1})\pi^{m-R_m}(1-\pi)^{R_m}} = \frac{\nu_{i,1}}{\nu_{i,1} + (1-\nu_{i,1})p^{2S_m-m}} \tag{2}$$

where $p = (1-\pi)/\pi$ is the odds ratio. She then again chooses an action maximizing single-period expected payoffs, i.e. x_1 or x_2 depending upon whether $\mu_{i,2} > \nu_{i,2}$ or vice-versa. The process continues with each agent in the society choosing actions, obtaining information from her neighbors and updating beliefs, for all $t \geq 2$.

We remark that in the above specification, agents are employing a "bounded Bayesian" learning algorithm. While agents are using Bayes rule to modify prior beliefs to posterior ones using their information, *they do not attempt to infer the information and the behavior of all agents in the society, on the basis of their observations concerning their neighbors.* Moreover, they are also being boundedly rational in choosing a myopically optimal action in each period. It is important to motivate such behavior on the part of agents in our model. One justification might be that fully rational inference and strategic behavior generates a complex stochastic, dynamic game with imperfect monitoring, which is too difficult for us (and hence possibly for the agents also) to solve. On the other hand, bounded Bayesian behavior is a consistent learning rule which allows agents to incorporate past history in a simple fashion, while expected payoff maximization captures the idea that agents exhibit goal-seeking behavior.

Let $\{\xi_{i,t}\}_{i\in N, t\geq 1} = \{(\mu_{i,t}, \nu_{i,t})\}_{i\in N, t\geq 1}$ be the stochastic process of agent beliefs and $\{A_{i,t}\}_{i\in N, t\geq 1}$ be the stochastic process of actions. Let $(\Omega, \mathcal{F}, \mathrm{P})$ be the probability space on which the above processes are defined; a typical sample path in Ω is denoted by ω.[6] Since agents are employing Bayes rule in forming posterior beliefs, the Martingale Convergence Theorem may be applied to show convergence of beliefs.

Proposition 2.1 *There exist random elements $\{\xi_{i,\infty}\}_{i\in N}$ with each $\xi_{i,\infty} = (\mu_{i,\infty}, \nu_{i,\infty})$ such that with probability 1 we have $\lim_{t\to\infty} \xi_{i,t} = \xi_{i,\infty}$ for all $i \in N$.*

[6]The probability space may be constructed as an product space over the infinite sequences of payoff realizations over both actions and across all agents. See Appendix A in Bala and Goyal for details.

We are interested in the long-run choices of actions by the agents. In a connected society, since information can potentially flow over time from every agent to every other agent, we would expect some correlation across agents' actions in the long run. To formalize this idea, we define the concepts of social conformism and coexistence.

Definition 2.1 *Social conformism of actions is said to occur if, for almost all sample paths* $\omega \in \Omega$ *there exists a* $t(\omega)$ *and an action* $x(\omega)$ *such that* $A_{i,t}(\omega) = x(\omega)$ *for all* $i \in N$ *and all* $t \geq t(\omega)$*. If social conformism is violated, we say that actions exhibit coexistence.*

Coexistence of actions arises if there is a set of positive probability on which both actions are chosen infinitely often by members of the society. We can also define the notion of strong coexistence, which requires that in every time period, there are agents choosing different actions.

Definition 2.2 *The actions exhibit strong coexistence if, for all* ω *in a set* K *of positive probability, there exist agents* $i(\omega)$ *and* $j(\omega)$ *such that* $A_{i(\omega),t} \neq A_{j(\omega),t}$ *for all* $t \geq 1$*.*

The following result shows that whenever one (and only one) action is of the High type, there will be social conformism to that action. In addition, the posterior beliefs of each agent will also place point mass on the action being of the High type. Complete learning of the state of nature will also occur if both actions are of the Low type. In this case actions will exhibit coexistence.

Proposition 2.2 *Suppose that the society is connected. Let the true state be* $\theta \in \Theta = \{(H,H),(H,L),(L,H),(L,L)\}$*. (a) If* $\theta = (H,L)$ *then social conformism on action* x_1 *obtains. (b) If* $\theta = (L,H)$ *then social conformism on action* x_2 *obtains. (c) If* $\theta = (L,L)$ *then for almost all* $\omega \in \Omega$ *there exists an agent* $j(\omega)$ *such that* $A_{j(\omega),t} = x_1$ *infinitely often and* $A_{j(\omega),t} = x_2$ *infinitely often, i.e. there is coexistence of actions. Furthermore,* $\xi_{i,\infty}(\omega) = (1,0)$ *for all* $i \in N$*.*

The proof of this result, and subsequent ones is in the Appendix. Proposition 2.2 characterizes the limit outcomes for all states of nature other than (H,H). If the actions are of different quality types, then social conformism invariably occurs to the optimal action, independent of the structure of neighborhoods (information flows) in the society. Furthermore, while there is social coexistence of actions when the state of nature is (L,L), this phenomenon also holds independently of the neighborhood structure. We show that in contrast to the above cases, if both actions are of the High type, then the nature of information flows can influence the long run outcome. Specifically, depending upon the 'degree of connectedness' in a society, it is possible to sustain either social conformism or strong coexistence in the long run.

To show these possibilities, we consider a specific class of societies. For an integer $k \geq 1$, we shall suppose that the society N consists of $2k+2$ agents labelled as $1, \ldots, k$, α, β and $k+1, \ldots, 2k$. We shall refer to agents 1 to k and α collectively as the group N_1, and the rest of the agents as group N_2. Agents in N_1 have the same prior belief: $\xi_{i,1} = \xi_{j,1} = (\mu_1, \nu_1)$ for all i and j in N_1. Moreover, $\mu_1 > \nu_1$; since they are expected utility maximizers, they will all choose x_1 in the first period. Likewise, agents in the group N_2 all have the common belief given by (μ_2, ν_2) with $\nu_2 > \mu_2$; thus they choose x_2 in the first period. Finally, we suppose that the true state of nature is $\theta = (H, H)$.[7]

We consider the following neighborhood structure. All observation relations are assumed to be symmetric: $j \in N(i)$ implies $i \in N(j)$. There is public observability within Group N_1 (i.e. all agents in the group observe each other) and likewise within Group N_2. The observation links across the two groups is specified by a *degree of integration* parameter $\eta \in \{1, \ldots, k+1\}$. If the degree of integration is η this means that agents $\{1, \ldots, \eta-1, \alpha\}$ observe (and are observed by) agents $\{\beta, k+1, \ldots, k+\eta-1\}$.[8] Note that for any k, the case $\eta = k+1$ corresponds to full public observability while $\eta = 1$ is the minimum required for a society to be connected. Figure 1 shows a society with $k = 3$ (i.e. 8 agents) when $\eta = 1$, 2 and $k+1$. The first result derives conditions on the degree of integration that are sufficient for strong coexistence.

Proposition 2.3 *Suppose the society is as described above. If $k - \eta + 1 > \eta/(2\pi - 1)$ then there is a strictly positive probability that all agents in group N_1 will choose x_1 forever and all agents in group N_2 will choose x_2 forever, i.e. actions exhibit strong coexistence.*

The idea underlying the proof can be explained by considering the case of $\eta = 1$: First, applying standard arguments we identify a set of sample paths, which has positive probability, and on which each agent, acting in isolation, will choose his initial action forever. We then consider agent α's behavior when the society is linked in the manner described

[7] This structure of prior beliefs is assumed for expositional simplicity. Our arguments below also hold under more general conditions, with small modifications. In particular, the conclusions of Propositions 2.3 and 2.4 also obtain if all agents start with the same prior belief.

[8] This structure is motivated by examples of societies with well defined sub-groups – based on ethnic, linguistic, spatial or professional considerations – where interaction within a sub-group is very regular and at a high level but only a few members of a sub-group interact with members from another sub-group.

above. In each period, agent α observes the other members of N_1 choosing x_1 and also sees β, who is choosing x_2. If $k > 1/(2\pi - 1)$ then, with the help of the Law of the Iterated Logarithm, we can show that agent α will receive positive information about x_1 at a "faster" rate than positive information about x_2 and so will choose x_1 forever. Furthermore, all $i \in N_1 \backslash \alpha$ will choose x_1 forever since they only observe positive information on x_1 and no information on x_2. By symmetry, agent β will choose x_2 forever as will all members of N_2. Thus diversity occurs for the following reason: the agents at the "boundary" maintain their original choices because they are *more highly connected to agents who choose as they do as compared to those who choose differently.* If the information forthcoming from their own experience and experience of like-minded neighbors' actions is sufficiently positive it 'overcomes' the positive information concerning the other action coming from across the 'boundary', and the agents maintain their original decision.

It is worth noting some additional features of this result. On the set of sample paths identified in Proposition 2.3, agents α and β observe actions x_1 and x_2 infinitely often: by the law of large numbers, they will learn the true state in the long run (i.e. their beliefs will converge to point mass on state (H, H)). However, the remaining agents observe only either x_1 or x_2 and therefore do not learn the true state (more specifically, their beliefs concerning the action they observe will converge to 1 while their beliefs about the type of the other action will remain unchanged). Thus it is possible for some agents in a connected society to learn the truth while others simultaneously do not! Secondly, the above result also exhibits "path-dependency" since there exists another set of sample paths having positive probability such that one action will become extinct in finite time. The proof of this statement is not difficult and is omitted.

By way of contrast, we now consider the case of a fully integrated society, i.e. with $\eta = k + 1$, for which the above proposition does not apply. Our analysis is summarized by the following result.

Proposition 2.4 *Let the society be as described above and suppose that $\eta = k + 1$, so that there is full public observability. Then social conformism obtains.*

The intuition for this result is as follows. Suppose that an agent with prior $\xi = (\mu, \nu)$ receives new information of S_n successes in n trials with x_1 and R_m successes in m trials with

x_2 from his neighbors. Let his posterior belief computed according to (2) be $\xi' = (\mu', \nu')$. He will choose x_1 in the next period if $\mu' > \nu'$ or

$$\frac{\mu}{\mu + (1-\mu)p^{2S_n - n}} > \frac{\nu}{\nu + (1-\nu)p^{2R_m - m}} \Rightarrow p^{2(S_n - R_m) + m - n} < \frac{\mu(1-\nu)}{\nu(1-\mu)}. \tag{3}$$

where $p = (1-\pi)/\pi$. Given the rules of the process, the random variable $L \equiv 2(S_n - R_m) - n + m$ summarizes the manner in which new information obtained from an agent's neighbors affects his action. In particular, equation (3) implies that for 'high' values of L, both groups of agents will choose action x_1, for 'low' values of L both groups of agents will choose action x_2, while for intermediate values of L, group 1 will choose action x_1 while group 2 chooses action x_2. The process L can be represented as a stationary Markov Process; the proof then follows from the observation that for high (low) values of L, the process is a random walk with a positive drift (negative drift), while for intermediate values it is a symmetric random walk.

We have so far derived sufficient conditions on the degree of integration for coexistence to obtain and also shown that when a society is fully integrated then social conformity obtains. We now examine how the probability of coexistence varies in relation to the parameter of integration, with the help of simulations. As a check on the robustness of our analysis, we also consider the case of actions with normally distributed outcomes. These simulations are summarized in Figures 2 and 3, respectively. Our simulations were carried out for a society with $k = 9$ (total number of agents is 20). The figures suggest that there exists an inverse relationship between the degree of integration and the likelihood of coexistence. The intuition for this is that as η increases, the information set of agents on the 'boundary' becomes more alike, making different optimal choices by them less likely. Thus the agents who make the 'across-group' observations (such as α and β) choose the same actions. This allows information about both actions to flow into a sub-group thereby precipitating a breakdown in the 'information barrier' separating N_1 and N_2 and eventually leading to social conformism. The simulations also suggest that for fixed η, the likelihood of diversity increases as π and the value of the mean (in the normal distribution case, keeping standard deviation constant) decreases. Lowering π or the value of the mean in the normal case both have the effect of decreasing the informativeness of action x_1, which allows the influence of the initial differences in priors of the two groups N_1 and N_2 to survive more easily.

3 Conclusion

We study a social learning model with two technologies of unknown quality, and ask whether society will herd on to one technology (social conformism), or whether both can coexist in the long run. If one technology yields higher expected payoff, social conformism will occur. However, if both technologies are of high quality, then the structure of information flows in the society determines whether conformism or coexistence arises. Specifically, coexistence arises if information-sharing occurs more among agents having similar prior beliefs, than across such groups. By contrast, if there is uniform information sharing across agents, only one technology will prevail in the long run.

4 Appendix

Proof of Proposition 2.2: Lemma 3.1 and Lemma 3.2 in Bala and Goyal (1993) state that conditional on the true state, for every agent $i \in N$ the expected payoff $E[A_{i,t}; \xi_{i,t}]$ converges almost surely to a limit random variable $U_{i,\infty}$, where $U_{i,\infty}$ is the expected payoff (conditional on the true state) from any action chosen infinitely often. Furthermore, Theorem 3.2 there shows that in a connected society, the limits $U_{i,\infty} = U_{j,\infty}$ are the same almost surely for all agents i and j in N. In our context the above results imply that if $\theta = (H, L)$ then (i) almost surely, each agents will be choosing a single action x all but finitely often (ii) almost surely, x is the same action for all agents in N. We now see from the law of large numbers that if an agent observes the outcomes associated with some action infinitely often, his belief will converge to point mass on the true type of the action. Fix a sample path ω where the law of large numbers holds. If all agents choose x_1 only finitely often on ω, then $\mu_{i,\infty}(\omega) > 0$ and $\nu_{i,\infty}(\omega) = 0$ for all $i \in N$. However, for all sufficiently large t this implies $\nu_{j,t}(\omega) < \mu_{j,t}(\omega)$ for some agent j. Since agents are expected payoff maximizers, j will then choose x_1 almost always on ω, contradicting our supposition. Statement (a) now follows from (i) and (ii). For (b), a similar argument shows that if all agents choose the same action (say x_1) from some time onwards on a sample path ω, then $\mu_{i,\infty}(\omega) = 0$ and $\nu_{i,\infty}(\omega) > 0$. For sufficiently large t this contradicts expected payoff maximization. Hence there is some agent who will eventually choose x_2. By symmetry, and the fact that N is finite, there must exist some $j(\omega)$ who chooses both x_1 and x_2 infinitely often. Statement (b) now follows.

□

Proof of Proposition 2.3: Let $N_1' = \{j \in N_1 | N(i) = N_1\}$ denote the set of agents who are linked only to members of their own group, and let $N_1^{''} = \{i \in N_1 | N(i) \cap N_2 \neq \phi\}$ denote the set of agents who are also linked to members of the other group. Similarly define $N_2' = \{i \in N_2 | N(i) = N_2\}$ and $N_2^{''} = \{i \in N_2 | N(i) \cap N_1 \neq \phi\}$. Note that $|N_1^{''}| = |N_2^{''}| = \eta$; also recall that $\theta = (H, H)$ is the true state. In what follows, we shall let $\mathrm{P}^\theta(\cdot)$ denote probabilities conditional on the state of nature.

For any agent $i \in N$, let S_n^i (R_m^i) denote the number of successes when agent i chooses action x_1 (x_2) for n (m) time periods. Note that since $\mu_1 > \nu_1$, every $i \in N_1$ chooses x_1 at time $t = 1$. Assume that every agent $j \in N_1^{''}$ chooses x_1 in each of the periods $t = 1, \ldots, \bar{n}$, where $\bar{n}$ is some positive integer. A sufficient condition for $i \in N_1'$, to continue to choose only x_1 in every period $t = 1, \ldots, \bar{n}$ then follows from (3):

$$p^{2(\sum_{j\in N_1^{''}} S_t^j + \sum_{j \in N_1'} S_t^j) - (k+1)t} < \frac{\mu_1(1-\nu_1)}{\nu_1(1-\mu_1)}. \tag{A.1}$$

for every $t = 1, \ldots, \bar{n}$. Note that $\mu_1 > \nu_1$ implies that the right hand side of (A.1) is strictly greater than one, and that $0 < p < 1$. Thus a simpler and stronger sufficient condition than (A.1) is that for every $t = 1, \ldots, \bar{n}$

$$p^{2(\sum_{j\in N_1^{''}} S_t^j + \sum_{j \in N_1'} S_t^j) - (k+1)t} < 1 \Rightarrow \sum_{j\in N_1^{''}} S_t^j + \sum_{j\in N_1'} S_t^j \geq \frac{(k+1)t}{2}. \tag{A.2}$$

assuming as before that agent every $j \in N_1^{''}$ chooses x_1 in each time period up to $\bar{n}$. Since (H,H) is the true state, for each agent i and time t the random variable $2S_t^i - t$ is the sum of independent and identically distributed random variables of the form $2Z - 1$ where Z has a Bernoulli distribution with parameter $1/2 < \pi < 1$. Since $E[Z] = 2\pi - 1 > 0$, it follows from the standard theory of random walks that $\mathrm{P}^\theta(\{2S_t^i - t \geq 0 \text{ for all } t = 0, 1, \ldots\}) > 0$. Fix $\epsilon > 0$ and for $t \geq 3$ define $\psi(\epsilon, t) = (1+\epsilon)\sqrt{2t\pi(1-\pi)\log\log t}$. For $t \geq 3$, let E_t^i be the event $E_t^i = \{S_n^i \in [n\pi - \psi(\epsilon, n), n\pi + \psi(\epsilon, n)] \text{ for all } n \geq t\}$.

Note that the sets E_t^i are increasing in t. The law of the iterated logarithm (see Billingsley 1986) implies that $\mathrm{P}^\theta(\bigcup_{t=3}^\infty E_t^i) = 1$.. It follows from the previous equations that there is some T such that $\mathrm{P}^\theta(\{2S_t^i - t \geq 0 \text{ for all } t = 0, 1, \ldots\} \cap E_T^i) > 0$. Let E^i denote the above event, i.e. $E^i = \{2S_t^i - t \geq 0 \text{ for all } t = 0, 1, \ldots\} \cap E_T^i$. The set E^i is such that agent i's observations will satisfy $S_t^i \geq t/2$ for every t and also from T onwards, S_t^i will be within the bounds prescribed by the law of the iterated logarithm. Note that given that the previous equation holds for T it also holds from $T' > T$ since the sets $\{E_t^i\}$ are increasing. Thus we shall assume, without loss of generality, that T also satisfies the condition

$$T \geq \inf \Big\{t \mid ((k-\eta+1)(2\pi-1)-\eta)\frac{t}{2} - (k-\eta+1)\psi(\epsilon,t) > 0\Big\}. \tag{A.3}$$

Since $k-\eta+1 > \eta/(2\pi-1)$ by assumption, the term $(k-\eta+1)(2\pi-1)-\eta$ is positive. As the term $(k-\eta+1)\psi(\epsilon,t)$ in (A.3) is of the order of $t^{1/2}\log\log t$ it is eventually dominated by $((k-\eta+1)(2\pi-1)-\eta)t/2$. Hence, T in (A.3) will be finite. In addition, for all $t \geq T$ the expression $((k-\eta+1)(2\pi-1)-\eta)t/2-(k-\eta+1)\psi(\epsilon,t) > 0$ as well.

We now consider the situation of agent $i \in N_1''$ given that the rest of N_1 choose only x_1 up to time $\bar{n}$. We shall assume that agent $j \in N_2''$ chooses only x_2 in each of the $\bar{n}$ periods, and later isolate a set of sample paths where this will in fact be true. Recall that R_t^j denotes the number of successes that agent β obtains in t trials from using x_2. Using (3) and the arguments underlying (A.1) and (A.2) again, a sufficient condition for agent $i \in N_1''$ to choose x_1 for each $t = 1,\ldots,\bar{n}$ is

$$\sum_{j\in N_1''} S_t^j + \sum_{j\in N_1'} S_t^j \geq \sum_{j\in N_2''} R_t^j + \frac{(k-\eta+1)t}{2}. \tag{A.4}$$

We now choose the set of sample paths for agent $j \in N_1''$ which satisfy the following requirements:

$$\bar{E}^j \;=\; \{S_T^j = T,\; S_t^j \geq T + (t-T)/2,\ \text{for all } t > T\}. \tag{A.5}$$

Thus, for sample paths in $\bar{E}^j$, agent $j \in N_1''$ gets T successes in the first T trials with x_1 and subsequently gets a success rate of at least 50 percent in the remaining periods. Clearly $\mathrm{P}^\theta(\bar{E}^j) > 0$.

Next, we consider an agent $j \in N_2$. By definition of the set N_2, it follows that all $j \in N_2$ choose action x_2 in $t = 1$. Assuming that $j \in N_2''$ chooses x_2 in all periods up to $\bar{n}$, using arguments as above, it follows that a sufficient condition for each agent $i \in N_2'$ to choose x_2 for each $t = 1,\ldots,\bar{n}$ is :

$$\sum_{j\in N_2''} R_t^j + \sum_{j\in N_2'} R_t^j \geq \frac{(k+1)t}{2}. \tag{A.6}$$

For agents $i \in N_2'$ consider the sample paths $\{F_n^i\}$ defined as $F_t^i = \{R_n^i \in [n\pi - \psi(\epsilon, n), n\pi + \psi(\epsilon, n)]$ for all $n \geq t\}$. By symmetry with the situation of N_1, it follows that the event F^i defined as $F^i = \{2R_t^j - t \geq 0$ for all $t \geq 0\} \cap F_T^i$, has strictly positive probability. Lastly, we define the set $\bar{F}^j$ corresponding to $\bar{E}^j$ where (A.5) will be true for $j \in N_2^{''}$ in place of $j \in N_1^{''}$ and x_2 instead of x_1. We now consider the set of sample paths

$$\bigcap_{j \in N_1'} E^j \cap \bigcap_{j \in N_1^{''}} \bar{E}^j \cap \bigcap_{j \in N_2^{''}} \bar{F}^j \cap \bigcap_{j \in N_2'} F^j. \tag{A.7}$$

By construction, the above event has strictly positive probability. We claim that on this set of sample paths every $i \in N_1$ chooses x_1 and every $j \in N_2$ chooses x_2 forever. We first consider the situation for $t = 1, \ldots, T$. For any $i \in N_1'$, condition (A.2) is clearly satisfied, since on our choice of sample paths, $\sum_{j \in N_1'} S_t^j \geq (k - \eta + 1)t/2$ and $\sum_{j \in N_1^{''}} S_t^j = \eta t \geq t/2$ so that their sum is at least $(k+1)t/2$. Next consider an agent $i \in N_1^{''}$. Since $\sum_{j \in N_1^{''}} S_t^j + \sum_{j \in N_1'} S_t^j \geq \eta t + (k - \eta + 1)t/2$ and the right hand side of (A.4) equals this value (recall that $R_T^j = T$ for $j \in N_2^{''}$ on this set of sample paths) equation (A.4) also holds, ensuring that $j \in N_1^{''}$ will choose x_1 upto time T. By symmetry, we can show that any agent $j \in N_2$ chooses x_2 upto time T.

We now consider the position after time T. In this case, for each $i \in N_1'$, we have S_t^i in the interval $t\pi \pm \psi(\epsilon, t)$. Furthermore, for $j \in N_1^{''}$, S_t^j is at least equal to $T + (t - T)/2$ while R_t^j is at most equal to t. From (A.4) we have therefore

$$\begin{aligned} &\sum_{j \in N_1^{''}} S_t^j + \sum_{j \in N_1'} S_t^j - \Big\{ \sum_{j \in N_2^{''}} R_t^j + \frac{(k - \eta + 1)t}{2} \Big\} \geq \\ &\geq \eta \Big\{ T + \frac{(t - T)}{2} \Big\} + (k - \eta + 1)\Big\{ t\pi - \psi(\epsilon, t) \Big\} - \Big\{ \eta t + \frac{(k - \eta + 1)t}{2} \Big\} \\ &= \frac{\eta T}{2} + \Big\{ (k - \eta + 1)(2\pi - 1) - \eta \Big\} \frac{t}{2} - (k - \eta + 1)\psi(\epsilon, t). \end{aligned} \tag{A.8}$$

By our choice of T in (A.3) the last expression above is always non-negative. It follows that provided that every $i \in N_1'$ chooses x_1, and $j \in N_2^{''}$ always chooses x_2, any agent $j \in N_1^{''}$ will always choose x_1. We next show agents in N_1' will continue to choose x_1 after time T given that an agent $j \in N_1^{''}$ chooses x_1 forever. This requires condition (A.2) to hold. However,

$$\sum_{j \in N_1^{''}} S_t^j + \sum_{j \in N_1'} S_t^j \geq \eta \Big\{ T + \frac{t - T}{2} \Big\} + \frac{(k - \eta + 1)t}{2} \geq \frac{(k + 1)t}{2}. \tag{A.9}$$

so (A.2) continues to be satisfied. By symmetry of the agents N_2 to the agents N_1, analogous arguments establish that every agent in N_2 will always choose x_2.

$\square$

Proof of Proposition 2.4: Agents of group 1 all have the same prior, given by (μ_1, ν_1), as well as the same sources of information. Their decision problem is thus identical and is expressed in the inequality (3). This implies in particular, that there exists a number I_1 such that $2(S_n - R_m) + m - n > I_1 \Rightarrow \mu_1' > \nu_1'$ while $2(S_n - R_m) + m - n \leq I_1 \Rightarrow \mu_1' < \nu_1'$.

Similar calculations suggest that for group 2 agents, with priors (μ_2, ν_2), there exists a number I_2 such that, $2(S_n - R_m) + m - n \geq I_2 \Rightarrow \mu_2' > \nu_2'$ while $2(S_n - R_m) + m - n < I_2 \Rightarrow \mu_2' < \nu_2'$.

Moreover, since $\mu_1 > \nu_1$ and $\nu_2 > \mu_2$, it follows that

$$\frac{\mu_1(1-\nu_1)}{\nu_1(1-\mu_1)} > \frac{\mu_2(1-\nu_2)}{\nu_2(1-\mu_2)}. \tag{A.10}$$

and this implies that $N_1 \leq N_2$.

The configuration of actions at any point t thus depends on the value of $2(S_t - R_t) + m_t - n_t$, where S_t and R_t refer to the number of successes with actions x_1 and x_2, and n_t and m_t refer to the number of times actions x_1 and x_2 have been taken until time period t, respectively. Let $L_t = 2(S_t - R_t) + m_t - n_t$. The proof consists of showing that on (almost) every sample ω there is a point $\tau(\omega)$ such that either $L_t \geq I_2$ for $t \geq \tau(\omega)$ or $L_t \leq I_1$ for $t \geq \tau(\omega)$. This establishes that all agents try *only* one of the two actions x_1 or x_2 after a finite point, with probablity 1.

The process L_t can be represented as a stationary Markov Chain as follows: For $L_t \geq I_2$, all agents choose action x_1 and so the process is defined as follows:

$$L_{t+1} = L_t + 2S_{2k+2} - (2k+2). \tag{A.11}$$

where S_{2k+2} denotes the number of realizations of 1 in $2k+2$ trials with action x_1. Let $W_1 = 2S_{2k+2} - (2k+2)$. The transition probablities are determined by the true state of the world. Under our hypothesis, since the true state is (H,H), it follows that $E[W_1] > 0$. Thus in this region, L_t is a random walk with a positive drift.

Likewise, for $L_t \leq I_1$, all agents choose action x_2 and so the process is defined as follows:

$$L_{t+1} = L_t + (2k+2) - 2R_{2k+2}. \qquad (A.12)$$

where R_{2k+2} denotes the number of realizations of 1 in $2k+2$ trials with action x_2. Let $W_2 = (2k+2) - 2R_{2k+2}$. Since true state is (H,H), it follows that $E[W_2] < 0$. Thus in this region L_t is a random walk with a negative drift.

Finally, for $I_1 < L_t < I_2$, since group 1 agents choose action x_1 and group 2 agents choose x_2, the process is defined as follows:

$$L_{t+1} = L_t + 2(S_{k+1} - R_{k+1}). \qquad (A.13)$$

where S_{k+1} denotes the number of realizations of 1 in $k+1$ trials with action x_1 and R_{k+1} denotes the number of realizations of 1 in k+1 trials with action x_2. Let $W_0 = 2(S_{k+1} - R_{k+1})$. Since true state is (H,H), it follows that $E[W_1] = 0$. Thus in this region, L_t is a symmetric random walk.

¿From equations (A.11)-(A.13) it is clear that the process L_t is defined on the set of even integers, $S = \{\cdots, -4, -2, 0, 2, 4, \cdots\}$. Then $S = S_0 \cup S_1 \cup S_2$. We have now completed the description of the Markov Chain.

We note (from equations (A.11)-(A.13)) that $\mathrm{P}^\theta(L_{t+1} = L_t + 2) > 0$ and $\mathrm{P}^\theta(L_{t+1} = L_t - 2) > 0$. This immediately implies that $\{L_t\}$ is an irreducible Markov Chain. Applying Theorem 8.3 in Billingsley (1985) we conclude that the process L_t must be either persistent or transient.

The next step of the proof shows that $\{L_t\}$ is not persistent. We need some additional notation for this: Let $S_0 = \{I_1, I_1+2, I_1+4, \cdots, I_2\}$, $S_1 = \{I_2, I_2+2, I_2+4, \cdots\}$ and $S_2 = \{I_1, I_1-2, I_1-4, \cdots, I_2\}$. Then $S = S_0 \cup S_1 \cup S_2$. We have now completed Let $S_0 = \{I_1, I_1+2, I_1+4, \cdots, I_2\}$, $S_1 = \{I_2, I_2+2, I_2+4, \cdots\}$ and $S_2 = \{I_1, I_1-2, I_1-4, \cdots, I_2\}$. Then $S = S_0 \cup S_1 \cup S_2$. To show transience it suffices to show that there exist states $s, s' \in S$ such that the probablity of going from s to s' is less than 1. Consider a state $s \in S_2$ and a state $s' \in S_0$. Let $\gamma = \inf_{t \geq \tau}\{L_t > I_1 | L_\tau \leq I_1\}$ Next recall that $E[W_2] < 0$ and $E[W_2^+] < \infty$, where $W_2^+ = \max\{W_2, 0\}$. An application of Theorems 8.3.4, 8.2.5 and 8.2.4 in Chung (1974) then implies that $\mathrm{P}^\theta(\gamma < \infty) < 1$. In other words, the probability of going from state s to state s' (in finite time) is less than 1. This establishes that the process $\{L_t\}$ is transient. The transience implies, in particular, that any finite set of states is reached at most finitely many times, almost surely.

To complete the proof we only have to show that, along a generic sample path, L_t lies in only one of S_1 or S_2 after some finite point. Suppose not. Given the finitenes of S_0 (and the transience of L_t), this means that the process L_t lies in S_1 and S_2 infinitely often. We show that this constradicts the transience of the $\{L_t\}$ process. Define the set $\bar{S} = \{I_1 - (2k+2)\cdots, I_1, \cdots, I_2, \cdots, I_2 + (2k+2)\}$. Next observe that, given the range of values of W_1 and W_2, for L_t to cross over from S_1 to S_2 or vice-versa, it must necessarily pass through $\bar{S}$. Under our hypothesis, it therefore passes through the set $\bar{S}$ infinitely often. Since $\bar{S}$ is a finite set, this contradicts the transience of L_t and completes the proof. $\square$

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