
TRENDS IN PERIODIC AUTOREGRESSIONS

Richard PAAP

Econometric Institute
Erasmus University Rotterdam
Tinbergen Institute

1 INTRODUCTION

Standard econometric modelling of quarterly observed time series is limited to modelling the series after removing the seasonal pattern, which is usually based on seasonal adjustment methods like Census X-11. The assumption behind this strategy is that economic agents make their decision based on deseasonalised data. However, the work of Ghysels and Nerlove (1988) show that economic agents often have difficulties in separating the seasonal from non-seasonal fluctuations. Further, we observe changes in seasonal patterns of time series, which can be caused by *e.g.* changes in preferences for certain holiday seasons (from summer to winter) or the fact that clearance sales start earlier during recessions. This last argument shows that interaction can also exist between the stage of the business cycle and the seasonal pattern, see Canova and Ghysels (1994) for empirical evidence.

A time series model which inhabits an interaction between the stochastic trend and the seasonal pattern is the periodic integrated autoregressive model [PIAR], see Osborn (1988) for an economic justification of the model. We show that this model has a stochastic trend whose impact varies with the seasons. It is therefore not possible to analyze the trend and the seasonal pattern separately. The inclusion of deterministic elements in this model, like constants and trends, implies an exploding seasonal pattern in the long run in the sense that the annual growth rate in every quarter is different. In practice however, macroeconomic time series do not exhibit an exploding seasonal pattern, so we need restrictions on the constant and trend parameters, which prevent the quarters from diverging in the long run. In this article we derive parameter restrictions, which give rise to an equal (non-zero) annual drift term in every quarter.

The outline of the article is as follows. In section 2 we discuss the impact of the stochastic trend in first order PIAR models. Further, restrictions on the seasonal dummies and trend parameters, which imply zero or equal annual drift terms are derived. In section 3 the stochastic trend in quarterly observed real UK Imports is analyzed. Section 4 concludes.

2 THE MODEL

Periodic autoregressive models are characterised by periodically varying parameters in the autoregressive part of the model. In other words, the dynamic structure varies with the seasons. Formally, a periodic autoregressive model of order p [PAR(p)] for a quarterly observed time series, y_t , $t=1, \dots, n=4N$ can be written as

$$y_t = \sum_{s=1}^4 (\mu_s D_{st} + \tau_s D_{st} T_t + \sum_{i=1}^p \phi_{is} D_{st} y_{t-i}) + \varepsilon_t, \quad (1)$$

where ε_t is standard white noise, D_{st} represents seasonal dummies and T_t a trend variable defined as $T_t = [(t-1)/4] + 1$, where $[.]$ represents the entier function. An extension to periodically varying variances is straightforward but not considered here, see *e.g.* Osborn (1991) and Tiao and Grupe (1980).

To analyze the trend in PAR models we consider a simple PAR(1) model

$$y_t = \mu_s + \tau_s T_t + \phi_s y_{t-1} + \varepsilon_t. \quad (2)$$

As we have a different model in each season, it is convenient to rewrite (2) in a model with constant parameters. Let $Y_T = (Y_{1T}, Y_{2T}, Y_{3T}, Y_{4T})'$ consist of the quarterly observations y_t , stacked in an annual vector, *i.e.* Y_{sT} is the observation in season s in year T . The model in (2) can be written in the following multivariate representation,

$$A_0 Y_T = \mu + \tau T + A_1 Y_{T-1} + \varepsilon_T, \quad (3)$$

where

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 0 & 0 & 0 & \phi_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (4)$$

$\mu = (\mu_1, \mu_2, \mu_3, \mu_4)'$ and $\tau = (\tau_1, \tau_2, \tau_3, \tau_4)'$. The index T runs from 1 to $N = n/4$ and $\varepsilon_T = (\varepsilon_{1T}, \varepsilon_{2T}, \varepsilon_{3T}, \varepsilon_{4T})'$ represents a vector of stacked ε_t , see Gladyshev (1961).

The multivariate process in (3) is trend stationary if the root of the characteristic equation

$$|A_0 z - A_1| = z - \phi_1 \phi_2 \phi_3 \phi_4 = 0 \quad (5)$$

is inside the unit circle ($|\phi_1 \phi_2 \phi_3 \phi_4| < 1$), see Lütkepohl (1991). If however

displays the impact of accumulation of shocks, $\Sigma \varepsilon_{T-i}$, on each season s . As the rank of $A_0^{-1}A_1$ equals 1, the rank of $(A_0^{-1}A_1)A_0^{-1}$ must equal 1. This means that it can be written as an outer product of two vectors

$$(A_0^{-1}A_1)A_0^{-1} = \begin{pmatrix} 1 \\ \phi_2 \\ \phi_2\phi_3 \\ \phi_2\phi_3\phi_4 \end{pmatrix} (1 \quad \phi_1\phi_3\phi_4 \quad \phi_1\phi_4 \quad \phi_1). \quad (11)$$

If we multiply (11) with $\Sigma \varepsilon_{T-i}$ we obtain the impact of the accumulation of shocks. For the first quarter, this is given by

$$\sum_{i=1}^{T-1} \varepsilon_{1i} + \phi_1\phi_3\phi_4 \sum_{i=1}^{T-1} \varepsilon_{2i} + \phi_1\phi_4 \sum_{i=1}^{T-1} \varepsilon_{3i} + \phi_1 \sum_{i=1}^{T-1} \varepsilon_{4i}, \quad (12)$$

with similar expressions for the other quarters. As not all ϕ_s equal one, (12) implies that for periodically integrated time series shocks in different quarters have a different impact on the series and that the impact of a shock is also different for each quarter. So, the series contains a stochastic trend whose impact varies with the seasons. The stochastic trend and the seasonal pattern cannot be analyzed separately. In case all ϕ_s equal 1 [$y_t \sim I(1)$], (11) reduces to an outer product of two vectors of ones, which implies that shocks in different quarters have the same impact on each quarter as the periodic structure in (12) disappears.

From (2) it is difficult to see the role of the deterministic components due to the periodic structure of the model. It is therefore better to consider annual growth rates ($\Delta y_t = y_t - y_{t-4}$). Note that the annual growth rates in every quarter at time T are given by ΔY_T . By subtracting Y_{T-1} from Y_T given in (9), we obtain the annual growth rate at time T

$$\begin{aligned} \Delta Y_T &= A_0^{-1}(\tau + \varepsilon_T - \varepsilon_{T-1}) + (A_0^{-1}A_1)A_0^{-1}(\varepsilon_{T-1} + \mu + \tau(T-1)) \\ &= (A_0^{-1}A_1)A_0^{-1}\mu + (A_0^{-1} - (A_0^{-1}A_1)A_0^{-1})\tau + (A_0^{-1}A_1)A_0^{-1}\tau T + \eta_T \\ &= A\mu + (A_0^{-1} - A)\tau + A\tau T + \eta_T, \end{aligned} \quad (13)$$

where $A = (A_0^{-1}A_1)A_0^{-1}$ and $\eta_T = A_0^{-1}\varepsilon_T + (A_0^{-1} + (A_0^{-1}A_1)A_0^{-1})\varepsilon_{T-1} = A_0^{-1}\varepsilon_T + (A_0^{-1} + A)\varepsilon_{T-1}$. The annual drift term at time T is given by $A\mu + (A_0^{-1} - A)\tau + A\tau T$. To analyse the impact of the deterministic components, we will consider four cases.

Case I: $\tau_s = 0$ for all s , $\phi_s \neq 1$ for some or all s and $\phi_1\phi_2\phi_3\phi_4 = 1$.

We see from (13) that the annual drift term, hereafter denoted by $d = (d_1, d_2, d_3, d_4)'$

equals $A\mu$ or

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} = \begin{pmatrix} 1 & \phi_1\phi_3\phi_4 & \phi_1\phi_4 & \phi_1 \\ \phi_2 & 1 & \phi_1\phi_2\phi_4 & \phi_1\phi_2 \\ \phi_2\phi_3 & \phi_3 & 1 & \phi_1\phi_2\phi_3 \\ \phi_2\phi_3\phi_4 & \phi_3\phi_4 & \phi_4 & 1 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} \quad (14)$$

As the rank of the matrix $(A_0^{-1}A_1)A_0^{-1}$ equals 1, it is not possible to have an equal non-zero annual drift term in every season unless $\phi_s=1$ for all s , because $d_s=d_1$ for $s=2,3,4$ can only hold if $\phi_s=1$ due to the relation $d_s=\phi_s d_{s-1}$ with $d_0=d_4$. Using the decomposition of $(A_0^{-1}A_1)A_0^{-1}$ given in (11) it is easy to see that besides the trivial solutions $\mu=0$, the restriction for a zero annual drift term in every season s in case of no deterministic trends is given by

$$\begin{aligned} \mu_1 + \phi_1\phi_3\phi_4\mu_2 + \phi_1\phi_4\mu_3 + \phi_1\mu_4 &= 0 \text{ or} \\ \phi_2\phi_3\phi_4\mu_1 + \phi_3\phi_4\mu_2 + \phi_4\mu_3 + \mu_4 &= 0. \end{aligned} \quad (15)$$

Under this restriction (13) reduces to $\Delta Y_T = \eta_T$. Note that we only have to impose one restriction on the four seasonal dummies parameters to have a zero annual drift term in every season, due to the restriction $\phi_1\phi_2\phi_3\phi_4=1$.

Case II: $\tau_s=0$ and $\phi_s=1$ for all s .

From (14), it is clear that in case $\phi_s=1$ for all s the annual drift term is the same in every quarter and equal to $\mu_1+\mu_2+\mu_3+\mu_4$, so we do not need to impose any restrictions on the μ_s parameters to obtain an equal annual drift term in every quarter.

Case III: $\tau_s \neq 0$ for all s , $\phi_s \neq 1$ for some or all s and $\phi_1\phi_2\phi_3\phi_4=1$.

Now, we consider the situation where the series y_t also contains a deterministic trend. In order to prevent the series Y_{sT} from diverging in the long run, the total impact of the deterministic trend over a year in every quarter has to be equal or zero. The total impact of the deterministic trend is given by $A\tau=(A_0^{-1}A_1)A_0^{-1}\tau$. As the rank of $(A_0^{-1}A_1)A_0^{-1}$ equals 1, it is easy to see that an equal non-zero annual impact of the deterministic trend in every quarter is impossible unless $\phi_s=1$ for all s where one uses the same line of reasoning as below (14). A zero annual impact of the deterministic trend in every quarter can be obtained by letting the variable T vanish from (13). This corresponds to the restriction $A\tau=0$, which can be simplified to

$$\phi_2\phi_3\phi_4\tau_1 + \phi_3\phi_4\tau_2 + \phi_4\tau_3 + \tau_4 = 0, \quad (16)$$

where we use the same arguments as below (14). Under (16) we have a different deterministic trend in every quarter whose total impact over one year is zero. Equation (13) reduces to $\Delta Y_T = A\mu + A_0^{-1}\tau + \eta_T$.

It is now also possible, contrary to case I, to have an equal non-zero annual drift term in every quarter which restricts the series Y_{sT} from diverging in the long run. Under (16) the annual drift term is given by $A\mu + A_0^{-1}\tau$. Using the definition of d (annual drift in case $\tau_s = 0$ for all s) in (14) and the restriction in (16), the annual drift terms can be written as

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ \phi_2 & 1 & 0 & 0 \\ \phi_2\phi_3 & \phi_3 & 1 & 0 \\ \phi_2\phi_3\phi_4 & \phi_3\phi_4 & \phi_4 & 1 \end{pmatrix} \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{pmatrix}. \quad (17)$$

Notice that due to restriction (16) the annual drift term in the fourth quarter equals d_4 . The restriction for an equal annual drift term can be derived by setting the annual drift of the first three quarters equal to the annual drift term in the fourth quarter, d_4 , which results in the following set of equations

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \phi_2 & 1 & 0 \\ \phi_2\phi_3 & \phi_3 & 1 \end{pmatrix} \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} d_4 \\ d_4 \\ d_4 \end{pmatrix}. \quad (18)$$

Using the fact that $d_s = \phi_s d_{s-1}$ and $d_0 = d_4$, the restrictions on τ_1, τ_2, τ_3 for an equal annual drift term are given by the solution of (18)

$$\tau_1 = (1 - \phi_1)d_4, \quad \tau_2 = (1 - \phi_2)d_4, \quad \tau_3 = (1 - \phi_3)d_4 \quad (19)$$

and restriction (16) (or $\tau_4 = (1 - \phi_4)d_4$), where d_4 is given by

$$d_4 = \mu_4 + \phi_4\mu_3 + \phi_3\phi_4\mu_2 + \phi_2\phi_3\phi_4\mu_1. \quad (20)$$

Notice that we only need four instead of six restrictions, due to the restriction of periodic integration ($\phi_1\phi_2\phi_3\phi_4 = 1$). Equation (13) reduces to $\Delta Y_T = d_4 e_4 + \eta_T$, where e_4 is a 4-dimensional vector of ones.

Case IV: $\tau_s \neq 0$ and $\phi_s = 1$ for all s .

In case ϕ_s equals 1 for all s the total annual impact of the deterministic trend, $A\tau$, is the same in every quarter and given by $\tau_1 + \tau_2 + \tau_3 + \tau_4$. The restriction of an equal non-zero drift term in the presence of a deterministic trend, whose total impact over a year is zero ($\tau_1 + \tau_2 + \tau_3 + \tau_4 = 0$), is not possible as (16) and (19) imply that τ_s has to be zero for all s to obtain an equal non-zero annual drift term. Straightforward algebra shows that this is also impossible if the total annual impact of the deterministic trend in every quarter is equal.

The derived restrictions on the constants and trends parameters, $\mu_1, \mu_2, \mu_3, \mu_4, \tau_1, \tau_2, \tau_3$ and τ_4 can easily be tested using *e.g.* F -tests. Under periodic integration, these F -tests asymptotically follow a standard F -distribution.

Although the analysis of constants and stochastic and deterministic trends in this section is limited to first order periodic autoregressions and quarterly observed data, extensions to higher order processes and monthly data are usually straightforward. This will however increase the analytical burden, see Franses and Paap (1994b) for a suggestion on how to proceed.

3 REAL UK IMPORT

As an illustration of the previous analysis we will look at the log of UK quarterly real Imports, i_t , 1955.1-1988.4 (see appendix of Osborn (1990) for a description of the series). Figure 1 shows a plot of the series. The series exhibits a trending pattern. Figure 2 shows the separate quarters I_{sT} of the Imports series. We see that the quarters have a common trending pattern, and that the ordering of the seasons changes over time, indicating a change in the seasonal pattern.

In Franses and Paap (1994a) a PAR model for this series is constructed, based on a model selection strategy evaluated in the same paper. First, the order of the PAR model is determined based on F -tests for the significance of extra lag parameters and model selection criteria, like the Akaike Information Criterion and the Schwarz Criterion. The order of the model turns out to be 1 for a model with and without seasonal deterministic trends. The unrestricted model with seasonal deterministic trends is

$$i_t = \mu_s + \tau_s (T_t/100) + \phi_s i_{t-1} + \varepsilon_t, \quad (21)$$

with parameter estimates

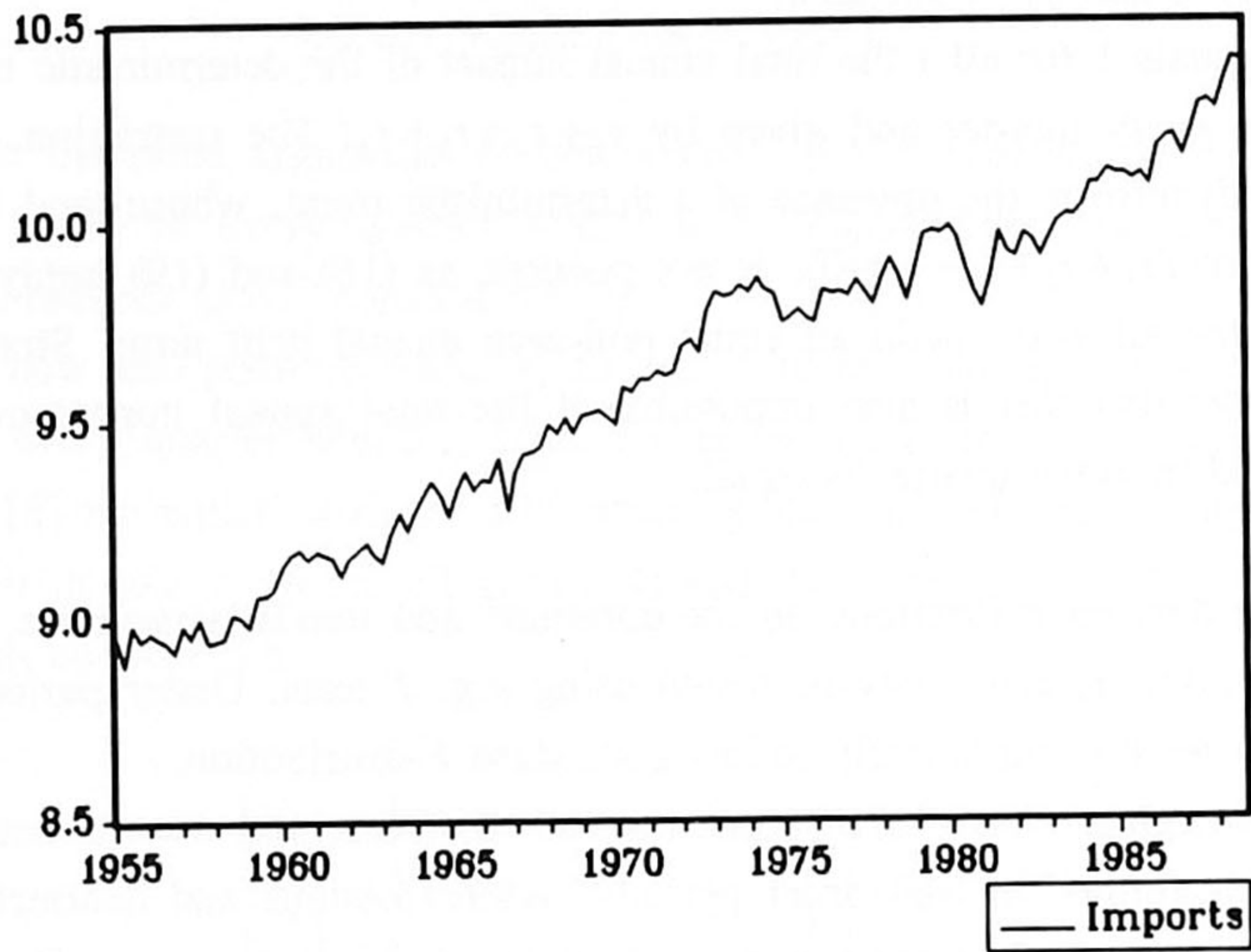


Figure 1. Log of the quarterly UK real Imports.

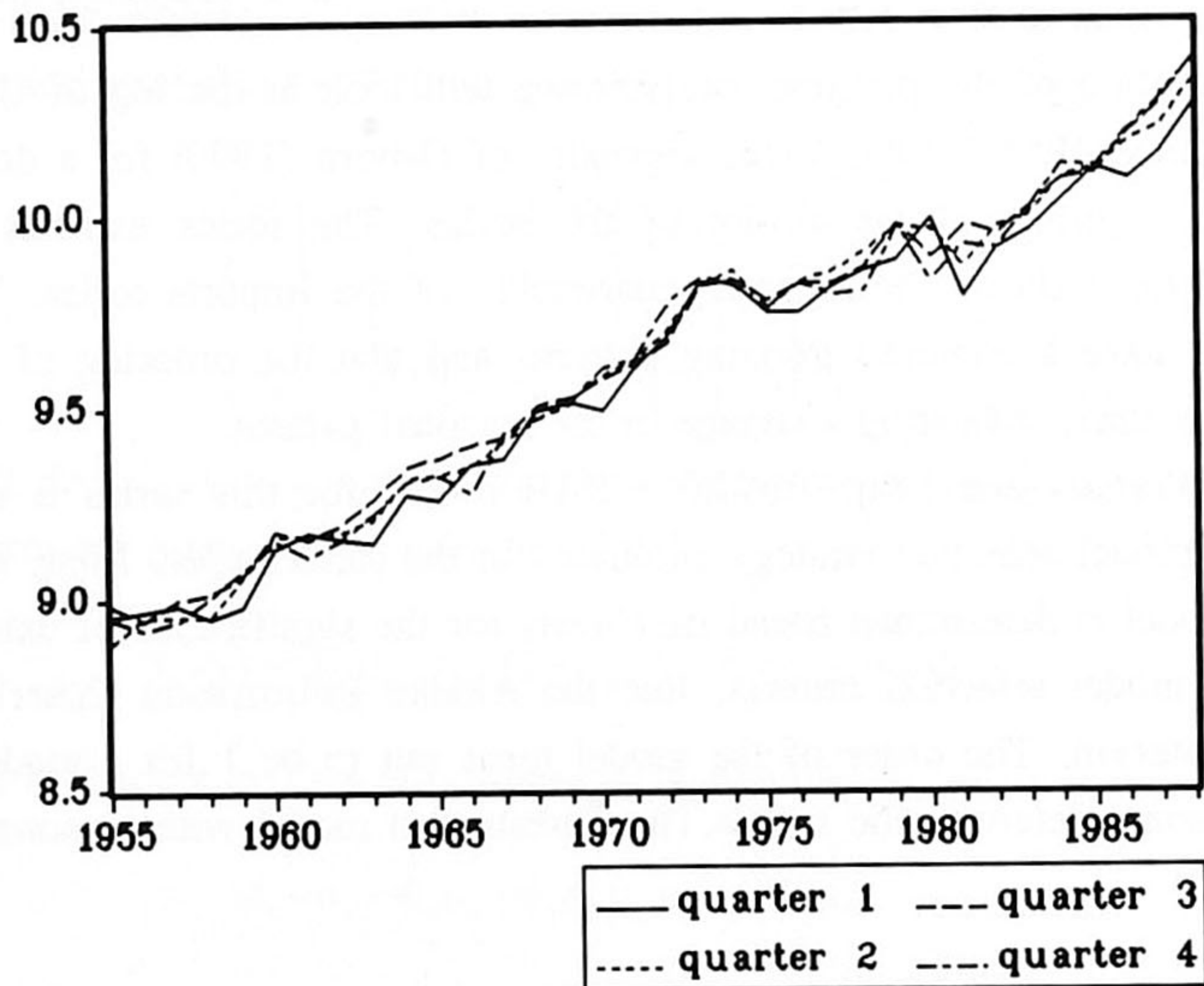


Figure 2. UK real Imports split up in separate quarters.

$$\begin{array}{cccc}
\mu_1=1.383 & \mu_2=2.487 & \mu_3=2.778 & \mu_4=-1.391 \\
(0.813) & (0.802) & (0.953) & (1.064) \\
\\
\tau_1=0.506 & \tau_2=1.314 & \tau_3=1.215 & \tau_4=-0.527 \\
(0.387) & (0.366) & (0.454) & (0.495) \\
\\
\phi_1=0.848 & \phi_2=0.720 & \phi_3=0.690 & \phi_4=1.152 \\
(0.092) & (0.090) & (0.107) & (0.119)
\end{array} \tag{22}$$

and standard errors between brackets. The $F(4,123)$ test for the hypothesis that $\tau_s=0$ for all s obtains a value of 5.713, which is significant at a 1% level under periodic integration and under periodic stationarity, see Franses and Paap (1994b) for a tabulation of the critical values for the different situations. In a second step we test whether the periodically varying autoregressive parameters are the same ($\phi_s=\phi$). If this is the case we can proceed with a non-periodic analysis of the time series and test for seasonal unit roots, see Hylleberg *et al.* (1990). An $F(3,123)$ test for $\phi_s=\phi$ in the model with deterministic trends equals 3.500 which is significant at a 5% level. In a model without deterministic trends we obtain an $F(3,127)$ test of 4.731 which is significant at a 1% level. These F -tests asymptotically follow an F -distribution, see Boswijk and Franses (1994). In both cases the autoregressive parameters differ significantly from each other. A test statistic for the restriction $\phi_1\phi_2\phi_3\phi_4=1$ within the model (21) is given by

$$BF = \text{sign}(\hat{\phi}_1\hat{\phi}_2\hat{\phi}_3\hat{\phi}_4-1)(n \ln(RSS_0/RSS_1))^{1/2}, \tag{23}$$

where RSS_1 and RSS_0 are the residual sum of squares of (21) with and without the restriction $\phi_1\phi_2\phi_3\phi_4=1$. Boswijk and Franses (1994) show that the BF test statistic follows the Dickey-Fuller distribution under the null hypothesis of periodic integration. For (21) we obtain a BF value of -3.282 which is significant at a 10% significance level. In case we do not include seasonal deterministic trends in (21) the BF statistic equals 0.105, which is not significant at any reasonable level. The parameter estimates of the restricted model ($\phi_1\phi_2\phi_3\phi_4=1$) with deterministic trend terms are

$$\begin{array}{cccc}
\mu_1=0.199 & \mu_2=1.188 & \mu_3=0.994 & \mu_4=-2.886 \\
(0.750) & (0.717) & (0.801) & (0.990) \\
\\
\tau_1=-0.049 & \tau_2=0.729 & \tau_3=0.373 & \tau_4=-1.216 \\
(0.359) & (0.329) & (0.384) & (0.461)
\end{array} \tag{24}$$

$$\begin{array}{cccc} \phi_1 = 0.982 & \phi_2 = 0.866 & \phi_3 = 0.891 & \phi_4 = 1/\phi_1\phi_2\phi_3 \\ (0.084) & (0.081) & (0.090) & \end{array}$$

with standard errors between brackets. It is clear that the trend parameters in the second and fourth quarter are significant. An $F(4,124)$ test for the joint significance of the four seasonal trend terms within the restricted model equals 2.942, which implies that the four trends are significant at a 5% level (the 95% fractile of an $F(4,124)$ distribution is equal to 2.776). In the final step in the model selection strategy we can perform an extra check to test in the PIAR(1) model whether the periodically varying autoregressive parameters, ϕ_s , are the same. An $F(3,124)$ test for the restriction $\phi_s = 1$ equals 5.582, which is significant at a 1% level.

To see the impact of the stochastic trend we look at the estimate of the matrix $(A_0^{-1}A_1)A_0^{-1}$, based on the estimates of ϕ_s in (23)

$$(A_0^{-1}A_1)A_0^{-1} = \begin{pmatrix} 1 & 1.155 & 1.296 & 0.982 \\ 0.866 & 1 & 1.122 & 0.850 \\ 0.772 & 0.891 & 1 & 0.758 \\ 1.018 & 1.176 & 1.320 & 1 \end{pmatrix}. \quad (25)$$

We see that shocks in the third quarter have the largest influence on the separate quarters of the series, as the elements of the third column are always larger than the corresponding elements of the other columns. Shocks in the fourth quarter have however the smallest influence. Shocks in any quarter have the largest impact on the fourth quarter of the series, as the elements of the last row of the matrix are larger than the corresponding elements of the other rows and the smallest impact on the third quarter.

It is now interesting to test whether the deterministic trend terms are relevant for the annual growth rates, so we test for the restriction (16) in the PIAR(1) model using an F -test. The result of the $F(1,124)$ test is 0.156 which is not significant at any reasonable level. Although the four deterministic trends terms are significant, the annual impact of the deterministic trend is zero. The last step is to test whether the annual drift terms in each season are the same. We test restrictions (16) and (19) using an $F(4,124)$ test. The outcome of the test is 3.583 which is significant at a 5% level, so the hypothesis of an equal annual drift term is rejected. The annual drift terms for the four quarters are however very small, respectively 0.021, 0.026, 0.027 and 0.023, so the differences between the quarters is small enough to prevent the quarters from diverging in the short run.

4 CONCLUSION

In this paper we have looked at the role of the trend in a first order periodic autoregressions. For the stochastic trend of a periodically integrated time series, shocks in different seasons have a different impact on each quarter. For integrated time series however, shocks in different seasons have the same impact on each quarter. The role of seasonal dummies and trends in periodic models is different than in non-periodic models. The implications of the deterministic terms is analyzed by looking at annual growth rates instead of less interpretable quarterly growth rates. We have shown that it is possible to have an equal annual drift term in the presence of deterministic trends. In that case the total impact of the deterministic trends over a year is zero.

The analysis is applied to the log of UK quarterly observed real Imports. This series turns out to be periodically integrated with a significant deterministic trend. Shocks in the third quarter have the largest impact on the individual quarters, while shocks in the fourth quarter have the smallest influence. Shocks in any quarter have the largest impact on the fourth quarter of the series, and the smallest impact on the third quarter. The total impact of the deterministic trend over a year turns out to be zero. An equal annual drift term in each season is however rejected.

ACKNOWLEDGEMENTS

This research was sponsored by the Economic Research Foundation, which is a part of the Netherlands Organisation for Scientific Research (N.W.O.). Comments from the editor and a referee are gratefully acknowledged.

REFERENCES

- Canova, F. and E. Ghysels, 1994, Changes in Seasonal Patterns: Are they Cyclical?, *Journal of Economic Dynamics and Control*, 18, 1143-1172.
- Boswijk, H.P. and P.H. Franses, 1994, Unit Roots in Periodic Autoregressions, T.I. report 94-4.
- Franses, P.H., 1994, Multi-step Forecast Error Variances for Periodically Integrated Time Series, Discussion paper 94-01, University of California, San Diego, to appear in *Journal of Forecasting*.

- Franses, P.H. and R. Paap, 1994a, Model Selection in Periodic Autoregressions, *Oxford Bulletin of Economics and Statistics*, 56, 421-439.
- Franses, P.H. and R. Paap, 1994b, Trends and Constants in Periodic Autoregressions, unpublished manuscript.
- Gladyshev, E.G., 1961, Periodically correlated random sequences, *Soviet Mathematics*, 2, 385-388.
- Ghysels, E. and M. Nerlove, 1988, Seasonality in Surveys. A Comparison of Belgian, French and German Business Tests, *European Economic Review*, 32, 81-99.
- Hylleberg, S., R.F. Engle, C.W.J. Granger and B. Yoo, 1990, Seasonal Integration and Cointegration, *Journal of Econometrics*, 44, 215-238.
- Lütkepohl, H., 1991, Introduction to Multiple Time Series Analysis, Springer-Verlag, Berlin.
- Osborn, D.R., 1988, Seasonality and Habit Persistence in a Life-Cycle Model of Consumption, *Journal of Applied Econometrics*, 3, 255-266.
- Osborn, D.R., 1990, A Survey of Seasonality in UK Macroeconomic Variables, *International Journal of Forecasting*, 6, 327-336.
- Osborn, D.R., 1991, The Implications of periodically varying coefficients for seasonal time-series processes, *Journal of Econometrics*, 48, 373-384.
- Tiao, G.C. and Grupe M.R., 1980, Hidden Periodic Autoregressive-Moving Average Models in Time Series Data, *Biometrika*, 67, 365-373.