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I urge Mr. Novick not to be cowed, and I applaud his plea for more communication and for more empirical work. Mathematical economists should, I think, take these criticisms to heart. The time has now arrived when most of the great obstacles to the testing of mathematical economic theories (inadequacy of data, inadequacy of statistical methods, lack of computational facilities) are being rapidly overcome; I am confident that the next decade will see a great advance in the testing of theories. But let it be remembered that mathematical methods and empirical research are not substitutes. On the contrary, empirical work can be useless and wasteful unless it is accompanied by good theory and good statistical methods, both of which are essentially mathematical in character.

IV. THE FUNCTIONS OF MATHEMATICAL TREATMENT

J. Tinbergen

1. It is certainly true to say that the present situation is unfortunate — the situation with respect to the use of mathematics in economic science — and improvement may be obtained as a consequence of a clearer understanding of the functions of mathematics. I very much welcome the attempt made by Dr. Novick, although I am inclined to put things somewhat differently. To what extent there is between us only a difference in wording and to what extent one of substance, I do not quite see. Therefore let me give my own view in my own words.

The functions of mathematical treatment in economic research may perhaps best be discussed on the basis of a breakdown into various elements of a *complete piece of econometric research.* Not all pieces of important economic analysis are by necessity also "complete": sometimes certain elements are absent, as a consequence of the special features of the problem handled. I do not want to say therefore that every contribution to economic science should show all the elements to be enumerated; but the function of mathematics becomes clearer if we consider this complete set of elements. The following elements will, in succession, have to be presented:

(i) a list of the phenomena to be included in the analysis will have to be given, in order clearly to delimit the realm of analysis and the degree of detail admitted;

(ii) symbols will have to be given, for the sake of clarity or shortness, if the number of phenomena exceeds a few — this being a question of "administration";

(iii) hypotheses or (partial) theories will have to be summed up that are assumed to determine the causal and other relations existing between the phenomena introduced — this being the element of economic theory;

(iv) these hypotheses will have to be given the form of equations, perhaps of a rather general form still, using function symbols yet unspecified to indicate relations which theory is not able to specify *a priori* — here we have to do with the mathematical formulation of the theories;

(v) a specification has to be given in the form of numerical determination of certain functions, based on observation of figures, including an indication of confidence intervals to certain numbers under certain assumptions — this being the element of statistical testing;

(vi) a combination of the thus specified partial theories has to be given in order to solve the problem set — the solution of the problem or the application of the theories used.

2. The functions mathematics may perform in this process are especially those indicated under (ii), (iv), (v), and (vi). They may somewhat more closely be considered now.

Function (ii), that of *notation*, is more important than is sometimes believed. As observed already, it is to be compared to what good administration is to a practical activity. Economic problems usually are multi-variate problems, problems in which many phenomena play a role, and the human mind has a limited capacity for memorizing. It is inefficient, if at all possible, to try to memorize without a special help. The help may be the more powerful the better devised the system of symbols is. This explains why people who are able to ab-

stract from formalities are sometimes nevertheless eager to obtain well-organized sets of symbols.

Function (iv), that of the *translation of economic hypotheses* or of economic (partial) theories, although also only preparatory and auxiliary, is very useful since it often compels us to state more precisely what we have in mind. It forces us to distinguish between demand, supply, and technical relations, to quote only a few; it requires a clear indication of which phenomena are assumed to affect demand, which supply, and the demand and supply of precisely what. Do we want to explain the demand for motor cars directly in terms of the attractiveness of a car or indirectly in terms of the attractiveness of its services; in what way precisely do fuel costs and taxes influence this attractiveness? Do we want to say that quantity supplied is a reaction to prices or that prices are seen as a reaction of suppliers to the quantity ordered? If speaking about the influence of interest rates on investment in stocks do we mean that the interest rate — and which one? — is a factor determining the total quantity held as a stock or the addition to the stock during a certain time period? Many more, and more modern and more complicated, examples could be given.

Function (v), that of *specification on the basis of observation*, is in fact a very complicated one; it represents, in this brief survey, the whole body of mathematical statistics. The central task in this element of mathematical technique is the calculation of probabilities on a number of assumptions. What is the probability of finding a certain set of observations (with given tolerances) if we assume that the economic theory of the phenomena measured is such and such — with numerical values for all sorts of elasticities, etc. — and the unexplained residuals have certain specified properties; and for what numerical values of the elasticities, etc., will this probability be largest? In the very simplest case, where we have only one phenomenon to explain, say sugar prices, and we try to do so by assuming that only sugar crops plus carryovers are relevant, what figure for demand elasticity should we take in order to make our observations most probable? Is the likeliest elasticity 0.3 or 0.35 or 0.25? The whole set-up of even such a simple problem can hardly be conceived of without mathematics.

Function (vi), finally, that of *combining partial theories* into one complete theory needed for the solution of the problem under consideration, takes the mathematical form of the solution of a system of equations, or, if such a solution is known already for a more general case, the application of the latter to the special case considered. This is the most typical function of mathematical economics. In cases of a more abstract piece of research it may take the form of a proof of a theorem, such as Gossen's laws or one of the existence theorems of Wald, i.e., the proof that under certain hypotheses there is one and only one equilibrium situation for a system of coherent markets.

3. After having enumerated the functions of mathematics in my own way, I would like to indicate *what functions in economic science it does not perform*; and why, in this connection, mathematical methods may sometimes be dangerous. It does not participate in the function indicated under (i), section 1: that of the enumeration of the phenomena to be included in the analysis. This is essentially a qualitative part of research, characterized by distinguishing different categories of economic concepts and by their exact definition: a typical task of the "literary" economist.

Mathematics has no task either in element (iii) of section 1, the functions of formulating hypotheses or (partial) theories. This formulation consists of the enumeration of such basic principles as the "economic principle," of institutional assumptions such as free competition or any other market strategy followed by the subjects considered, assumptions as to the production and cost functions relevant to the economy studied, the instruments of economic policy chosen, etc.

In addition to these two functions in which mathematics can not in principle even make a contribution, there may be special cases of the other functions where it may not be necessary to use mathematics since these functions can just as well be performed without it. I will come back to that situation in the next section.

First, a word may be said about certain dan-

gers of a mathematical treatment connected with the two functions of economic research just enumerated. If the analysis is carried out by people too enthusiastic for the mathematics involved they may either somewhat neglect those functions or they may make certain basic assumptions because they are easily treated mathematically. If by so doing they choose unrealistic assumptions they are actually not yielding a service to economics. Needless to say, the real great mathematical economists will not make such mistakes; but there are examples of engineers or physicists hunting for "analogies" between physics and economics and thereby biassing their theories. I would however like to add a remark that might be easily misunderstood and which I therefore hesitate to make: it is not always a disadvantage at first to investigate those cases which, although a little bit unrepresentative, are amenable to mathematical analysis; one may make discoveries of a more general character that prove to be useful later.

4. Next I propose to consider certain functions, already summed up in section 2, which *mathematics performs in competition with other methods or languages*. It evidently depends on the economic problem before us whether functions (ii), (iv), (v), and (vi) really need to be performed with mathematical help. In simple problems it will often not be necessary to use heavy mathematical "guns." The roundabout way which the introduction of symbols always implies may not pay: and there are also "economics of economics," as Goudriaan puts it. Even in cases where the use of mathematics is decidedly an advantage there is much to be said in favor of always using the simplest possible type, if only because the number of readers able to follow will then be a maximum. Being myself a mathematician of only modest knowledge, I often experience considerable difficulties when reading Cowles Commission stuff. The general recipe I venture to recommend here is that a new method or a new idea should always first be illustrated by the simplest conceivable case in which it presents itself and only afterward be treated in a general way. The general treatment is of course also needed in order to find out how far the method or idea brings us; but if you start with the simple case you will make it much easier to follow you. Where mathematics actually is in competition with other methods it should behave competitively: make itself as attractive and as efficient as possible.

In less simple cases the balance, in my opinion, quickly changes in favor of mathematics. Having even a restricted list of variables and a number of equations representing the mechanism discussed always means much more clarity, sharpness, and brevity — even for people who deny this. Of course there is a tendency for everybody to want it in a form just adapted to his state of knowledge of and familiarity with mathematics; I am coming back to this in section 7.

Contrary to what I understand Dr. Novick holds, I am of the opinion that the use of mathematical symbols also proves very useful even if applied to concepts that have not yet accurately been measured; we have every possibility of indicating at the same time the margins of error involved, as is usual in stochastic equations. A particular advantage is what I call the possibility of localization of certain influences. If it is maintained that, say, "the rate of discount has an influence on the cycle," we should be exactly informed about the place at which, i.e., the equation or equations in which, that influence expresses itself. Mathematical treatment forces us to so specify, whereas with non-mathematical treatment there is a tendency to be less clear about it.

5. There are, however, also functions in which mathematics is the only way to solve a problem. This is most clearly the case with the functions (v) and (vi), in more complicated problems. Specification of numerical values of parameters in a way is itself a mathematical process, although in the simpler cases it is so simple as to be accessible even to the layman. In the more complicated cases of mathematical statistical procedures there is no other way, and I do not think it has ever been proposed to estimate parameters by literary methods. The literary economist will consider this probably even to be outside the field of economics and not very much difference of opinion seems to exist on the necessity for mathematics here.

There is a second class of cases, however,

namely those within function (vi), for which it is just as true that sometimes mathematics is indispensable. To find the joint result of a number of partial theories or equations is not always possible by "reasoning." It is not correct even, in my opinion, to hold that every result of mathematical treatment can also be expressed in verbal form. It depends on what is meant exactly by this phrase. I tried to specify the questions involved in a paper read before the Royal Dutch Academy of Sciences [2] of which I may here briefly summarize the contents. By "reasoning" we understand in ordinary speech a sort of one-dimensional (or "one-way") logic, consisting of a succession of statements each of which can be proved with the aid of the foregoing. It is not always possible to solve a system of simultaneous equations by such "reasoning," i.e., to find each unknown in succession. This is even in general not possible and only possible if the system of equations is of what Wold calls the recursive type. In that case there will be one equation containing only one unknown and that unknown can thus be found; there will be a second equation containing that one and one further unknown; the latter can be found as a second step. And so all may be found in succession. This is not so for a system of simultaneous equations generally; and there is no equivalent to reasoning in such a case. Therefore it will not be possible to give a verbal deduction of the solution; it can only be tested afterward. In this sense it is not correct to maintain that mathematics does not add anything new or that the mathematical process can always be translated into ordinary speech. The results may be translated, but the process cannot be translated into "reasoning." It is another thing, of course, that every mathematical equation can always be given a verbal interpretation but that would not be very helpful in understanding the process.

6. There are also a number of *misunderstandings about mathematics*. Sometimes it is believed that only certain very simple and therefore "rigid" relations are representative by mathematics and that reality is more flexible, or however it may be expressed. This is to underestimate the power of mathematics: more advanced mathematics is able to express also much more complicated and flexible relations and partly to handle them. On the other hand it is sometimes forgotten that arguments against the most general types of mathematics are just arguments against science in general, i.e., against the assumption that we can understand connections between phenomena — in this case economic phenomena — in some general way. If determinacy — in whatever loose form — is not accepted at all, there is no economics: no mathematical economics and no literary economics. Perhaps there would remain economic novels; personally I would prefer other novels then.

7. May I finish with a few recommendations? As I said, it is unfortunate that different groups of economists do not understand each other at present. The reason is partly that certain of them express themselves "so mathematically" that they cannot be understood. To them my recommendation would be to use the simplest mathematics compatible with the problem they treat, and as much as possible always to start with a simple example. Another recommendation to some of them might be to switch over to empirical studies, since there is some over-production of theoretical work to which the factual basis is lacking.[3]

There is, however, also another reason for the lack of understanding between the two groups: it is the insufficient knowledge of mathematics with a number of economists. It has by now become clear (some think it was clear already half a century ago) that mathematics is an indispensable tool in modern economic analysis. The consequences should be drawn; fortunately they have in most universities already been drawn. And it is heartening to see how easily the younger generations of students are handling mathematical questions. The fear that the introduction of mathematics as an obligatory course would greatly reduce the number of economics students has not been confirmed.

There remains a natural division of labor

[2] J. Tinbergen, "In hoeverre kunnen economische stellingen zonder wiskunde worden bewezen?", Meded. Kon. Ned. Akad. v. Wet. afd. Lett.13 (1950), No. 10.

[3] Cf. J. Tinbergen: "Efficiency and Future of Economic Research," *Kyklos*, Vol. V (1953), 309.

between those more interested in qualitative and descriptive research and those more interested in quantitative and analytic work. Both have their places in economic science and the second group will need more mathematical tools than the first.

V. ON THE USE AND MISUSE OF MATHEMATICS IN PRESENTING ECONOMIC THEORY

D. G. Champernowne

Articles on economic theory are usually intended to explain to the reader what results may be expected to follow various policies or disturbances in given circumstances. The task of explanation is made difficult by the varying capacities and demands of the readers. If the theory is to be realistic, the given circumstances must reflect a great many of the complications of a real-world situation: an exhaustive account of these, whilst necessary for logical completeness, would be intolerably lengthy and boring for the reader who is well informed about the situation to which the theory is intended to apply. He can supply for himself, as obvious, many of the underlying assumptions about institutions and psychology which justify the reasoning in good non-mathematical articles on economic theory. But some readers will lack the experience or the docility to do this, and will either attempt to apply the theory to inappropriate circumstances, or will attack the writer for not giving an exhaustive account of his assumptions. Since proof is not always the most effective method of explanation, many writers are content to sketch their ideas in a manner which will enlighten sympathetic readers without attempting to convey complete conviction to anybody.

The cautious economic theorist, whose overriding ambition is never to appear wrong and yet to appear in print at all, has little scope beyond the discussion of economic models. These are shadows of the real situations so drastically simplified that they can be completely described and many of their workings exactly portrayed within the compass of a single article. The logic leading from the assumptions about the model to the conclusions about how it will behave can now be made rigorous and independent of the reader's knowledge of the real world. Such models serve the useful purpose of making it possible to reach complete agreement within reasonable time: occasionally a study of them will reveal that earlier disagreements have been entirely due to a difference of intuitive assumptions about the real world.

Unfortunately for the cautious theorist, his economic models will be judged according to the degree in which they appear to be relevant to the real world; so that in avoiding the appearance of being wrong, he may yet appear silly by publishing a long article whose relevance to any practical issue seems to be superficial. This danger of manufacturing mere "toys" is especially great since the assumptions which are most convenient for model-building are seldom those which are most appropriate to the real world.

Many would regard as the best and most important articles on economic theory those which reveal keen observation and judgment in choosing assumptions which accord well with facts and which are yet able to demonstrate fairly convincingly powerful conclusions of a simple nature, which suggest important analogues in the real world. They would regard the rigor of the logic and the exactness of the descriptions of the assumptions as secondary matters relating to their style rather than to their importance.

The ability to judge the relevance of an economic theory and its conclusions to the real world is but rarely associated with the ability to understand advanced mathematics. An important article on economic theory is therefore likely to be wasted unless it can be set out in prose supported by the most elementary mathematics. "It is obvious that there is no room in economics for long trains of deductive reasoning," [4] so that those economic models which are realistic and yet sufficiently simple to allow

[4] Marshall, *Principles of Economics*, 8th ed., Appendix D, p. 781, l.9.